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IDENTIFICATION OF THE INPUT  
DISTRIBUTION MATRIX FOR  
LINEAR DYNAMIC SYSTEMS  
OPERATING IN A STOCHASTIC  
ENVIRONMENT

by

Michael Raymond Polk



# United States Naval Postgraduate School



## THESIS

IDENTIFICATION OF THE INPUT DISTRIBUTION  
MATRIX FOR LINEAR DYNAMIC SYSTEMS  
OPERATING IN A STOCHASTIC ENVIRONMENT

by

Michael Raymond Polk

T132821

June 1969

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Identification of the Input Distribution  
Matrix for Linear Dynamic Systems  
Operating in a Stochastic Environment

by

Michael Raymond Polk  
Lieutenant (junior grade), United States Navy  
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Submitted in partial fulfillment of the  
requirements for the degree of

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1969

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~~TOP SECRET~~ P 683 C 1ABSTRACT

An algorithm for the identification of the input distribution matrix of a linear, stationary system operating in a stochastic environment is derived. The identification is accomplished by defining a set of autocorrelation functions for a noise element composed of a linear combination of the input distribution matrix elements and the random excitations of the system. Another possible identification method employing a Kalman filter is discussed and the problems associated with its derivation are presented. Results of computer simulations for both methods are included.

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## TABLE OF SYMBOLS

### A. VECTORS. (All underlined lower-case letters are vectors.)

$\underline{x}(t)$	Continuous representation of state vector
$\underline{x}_k$	Discrete representation of state vector
$\underline{y}_k$	State vector of the canonic system
$\underline{w}_k$	System excitation

### B. MATRICES. (All upper-case letters are matrices unless otherwise stated.)

A	Matrix describing the homogeneous system
B	Input matrix of the system
$\Phi(t)$	Homogeneous solution of the system
$\Phi$	State transition matrix of the discrete representation
$\Phi^*$	Canonic state transition matrix
$\Gamma$	Input distribution matrix of the system
$\Gamma^*$	Canonic input distribution matrix
Q	Covariance of the random excitation

### C. SCALAR QUANTITIES

$z_k$	Measurement of the system output
$v_k$	Additive measurement noise
n	System order
k	Time index

### D. OPERATORS

$x^T$	Denotes x transpose
$(.)^{-1}$	Denotes (.) inverse

$E[.]$	Denotes the expected value of $[.]$
$\Sigma$	Denotes a summation
$z$	Denotes the $z$ -transform variable

## I. INTRODUCTION

In recent years, an emphasis on plant identification for a system operating in a stochastic environment has developed in the field of automatic control. It is necessary to know the dynamics of the system before it may be effectively controlled. In order to know the system dynamics, a suitable mathematical model of the system must be found. For a system subjected to a random forcing function, finding a suitable mathematical model involves the identification of both the state transition matrix and the input distribution matrix for the random forcing function. Several methods have been proposed for the identification of the state transition matrix, but no one has proposed a method for identifying the input distribution matrix for systems with a single random input. The purpose of this investigation is to propose such a method. The problem may be defined as follows:

Given:

1. A system whose behavior may be defined by a set of linear, constant-coefficient, differential equations.
2. A statistical description of the excitation of the system.
3. A sequence of observations on the state vector.
4. An estimate of the state transition matrix of the system.

Problem:

Determine the input distribution matrix of the system.

Although this investigation is concerned only with systems with random forcing functions, the methods described may be extended to systems with both random and control inputs. The method of section



III may be extended by changing the form of the estimated state transition matrix as described by Lee [5]; and that of section IV, by including the effect of the control input in the filter equations as described by Kalman [3]. The problem is restricted to random forcing functions for simplicity.

## II. BASIC MATHEMATICAL MODEL

The investigation reported here starts with the assumption that there exists a set of  $n$  linear, constant-coefficient, differential equations of the form

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{w}(t) \quad (2.1)$$

which describe the dynamic behavior of a system subjected to a random forcing function,  $\underline{w}(t)$ . The solution of these equations is

$$\underline{x}(t) = \Phi(t-t_0) \underline{x}(0) + \int_{t_0}^t \Phi(t-\tau) B\underline{w}(\tau) d\tau \quad (2.2)$$

where

$$\Phi(t) = \mathcal{L}^{-1} [(sI - A)^{-1}].$$

Because the continuous system dynamics will be observed at discrete times, it is necessary to describe the system by discrete or difference equations. Introducing a sampling device of period  $T$  and a zero-order hold on the forcing function allows the system equations to be written

$$\underline{x}_{k+1} = \Phi(T)\underline{x}_k + \Gamma(T)\underline{w}_k, \quad (2.3)$$

where  $\underline{x}_k$  is the system output vector at a time  $kT$ ,  $\Phi$  is the  $n \times n$  discrete state transition matrix,  $\Gamma(T)$  is the  $l \times n$  input distribution matrix and  $\underline{w}_k$  is the sampled and zero-order held input vector at time  $kT$ . The components of the input vector  $\underline{w}_k$  are assumed to form a gaussian white sequence of zero mean and known variance during the period of system operation under investigation.

Observations made on the system output are assumed to be scalar linear combinations of the states of the system which are contaminated by additive gaussian white noise of zero mean and known variance. At

the  $k^{\text{th}}$  sampling instant the observation  $z_k$  is given as

$$z_k = Hx_k + v_k \quad (2.4)$$

where  $H$  is the  $1 \times n$  known measurement matrix and  $v_k$  is the scalar additive measurement noise. The system may then be represented by the block diagram of Figure 1.

In the general problem of identification of linear dynamic systems, it is necessary to estimate the elements of the state transition matrix before proceeding to identify the input distribution matrix. For the purposes of this investigation, this estimation is assumed done by using the method described by Lee [5]. The estimation of the state transition matrix by this method makes it necessary to change the system model to the so-called canonic form. Assuming that the system is observable and  $n$ -identifiable, the difference equation for the system model then becomes

$$y_{k+1} = \Phi^* y_k + \Gamma^* w_k \quad (2.5)$$

$$z_k = H^* y_k + v_k$$

where

$$y_{k+1} = Cx_{k+1}$$

$$C = \begin{bmatrix} H \\ H\Phi \\ \vdots \\ H\Phi^{n-1} \end{bmatrix} \quad (2.6)$$

$$\Phi^* = C\Phi C^{-1} = \begin{bmatrix} 0 & \vdots & & \\ 0 & & & I \\ \vdots & & & \\ \varphi_1 & \varphi_2 & \dots & \varphi_n \end{bmatrix}$$

$$\Gamma^* = C\Gamma = \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$H^* = [ 1 \ 0 \ 0 \ \dots \ 0 ]$$

The matrix is now the estimated state transition matrix obtained by using the method of Lee. The canonic system model can then be represented by the block diagram of Figure 2.

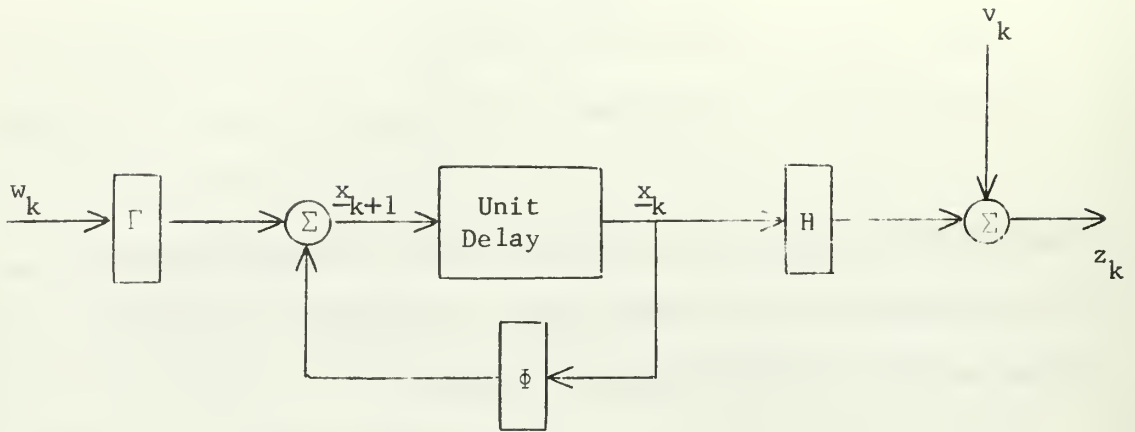


FIGURE 1. BLOCK DIAGRAM OF THE SYSTEM

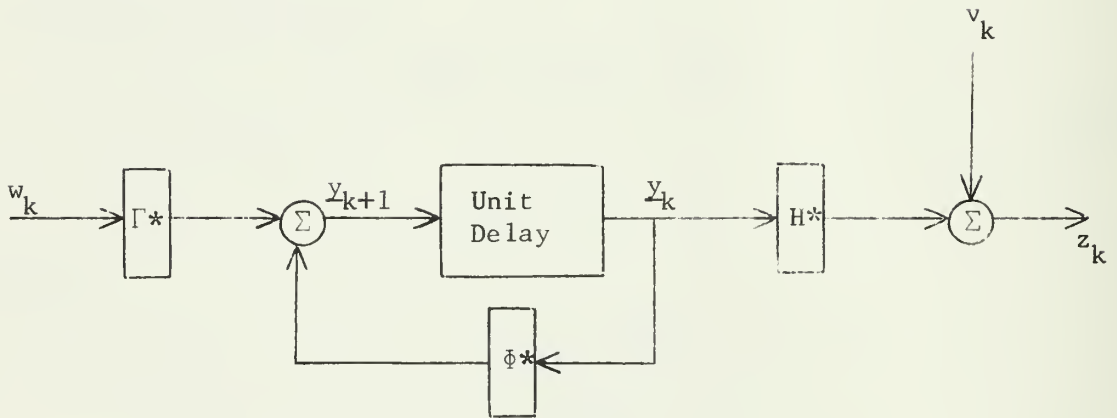


FIGURE 2. BLOCK DIAGRAM OF THE SYSTEM IN CANONIC FORM

### III. DERIVATION OF THE IDENTIFICATION ALGORITHM

Consider a system with canonic state equations of the form

$$\underline{y}_{k+1} = \Phi^* \underline{y}_k + \Gamma^* \underline{w}_k \quad (3.1)$$

$$z_{k+1} = H^* \underline{y}_{k+1} + v_k$$

where  $\underline{w}_k$  is assumed to be a scalar random forcing function. The  $z$ -transformed state equation becomes

$$z\underline{y}(z) - z\underline{y}(0) = \Phi^* \underline{y}(z) + \Gamma^* w(z). \quad (3.2)$$

Assuming that the initial conditions  $\underline{y}(0)$  are zero or that the identification process does not start until the initial conditions no longer affect the system output, the system may be represented by the signal-flow graph of Figure 3. From this signal-flow graph, the transfer function  $Z(z)/W(z)$  may be written as

$$\frac{Z(z)}{W(z)} = \frac{\frac{b_n}{z^n} + \frac{b_{n-1}}{z^{n-1}} \left(1 - \frac{\varphi_n}{z}\right) + \frac{b_{n-2}}{z^{n-2}} \left(1 - \frac{\varphi_n}{z} - \frac{\varphi_{n-1}}{z^2}\right) + \dots}{1 - \frac{\varphi_n}{z} - \frac{\varphi_{n-1}}{z^2} - \dots - \frac{\varphi_2}{z^{n-1}} - \frac{\varphi_1}{z^n}} \quad (3.3)$$

Since the elements of the  $\Phi^*$  matrix are assumed to be known, the problem can now be stated as finding the location of the zeros of the transfer function and thus the values of  $b_1, b_2, \dots, b_n$ .

From the transfer function, the difference equation relating the measured output  $z_k$  to the input  $w_k$  may be written as

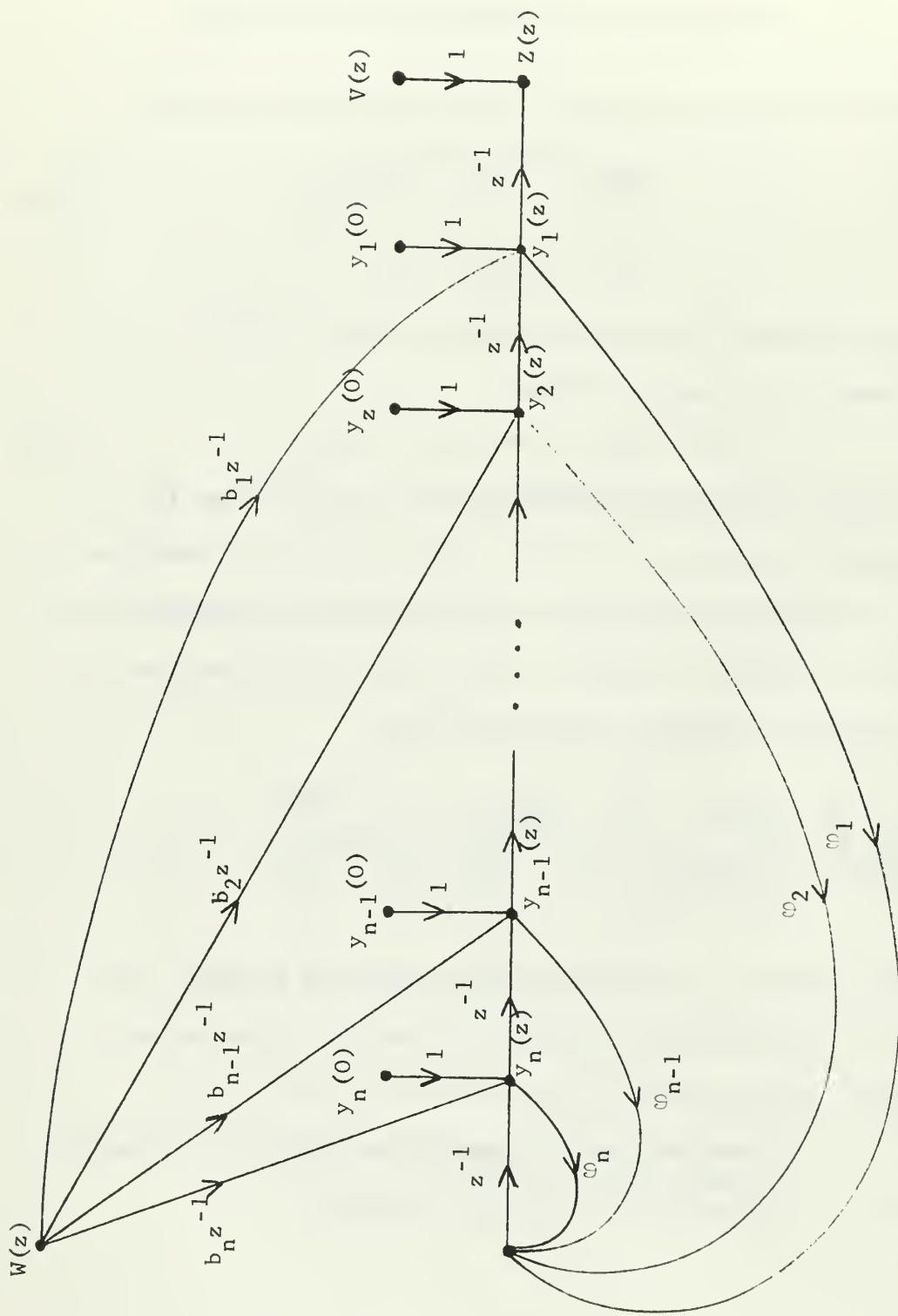


FIGURE 3. SIGNAL-FLOW GRAPH OF THE SYSTEM

$$\begin{aligned}
z_k = & v_k + \varphi_1 z_{k-n} + \varphi_2 z_{k-n+1} + \dots + \varphi_n z_k - b_1 w_{k-1} - (b_2 - \varphi_n b_1) w_{k-2} \\
& - (b_3 - \varphi_n b_2 - \varphi_{n-1} b_1) w_{k-3} \\
& - \dots - (b_{n-1} - \varphi_n b_{n-2} - \dots - \varphi_3 b_1) w_{k-n+1} \\
& - (b_n - \varphi_n b_{n-1} - \dots - \varphi_2 b_1) w_{k-n} .
\end{aligned} \tag{3.4}$$

Shifting indices by +1 and using vector notation, Equation (3.4)

becomes

$$z_{k+1} = \underline{\varphi}_k^T \underline{z}_k + \underline{n}_k^T \underline{d} + v_k \tag{3.5}$$

where

$$\begin{aligned}
\underline{z}_k &= \begin{bmatrix} z_{k-n+1} \\ z_{k-n+2} \\ \vdots \\ z_k \end{bmatrix} \\
\underline{\varphi} &= \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \end{bmatrix} \\
\underline{n}_k &= \begin{bmatrix} w_k \\ w_{k-1} \\ \vdots \\ w_{k-n+1} \end{bmatrix} \\
\underline{d} &= \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ -\varphi_n & & 1 & & \\ -\varphi_{n-1} & -\varphi_n & & 1 & \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ -\varphi_2 & -\varphi_3 & -\varphi_4 & \dots & 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}
\end{aligned} \tag{3.6}$$



Because the elements of both the vectors  $\underline{n}_k$  and  $\underline{d}$  are unknown, a single noise element is defined as

$$\Omega_k \stackrel{\Delta}{=} \underline{n}_k^T \underline{d} + v_k \quad (3.7)$$

where  $\Omega_k$  is a scalar and

$$\begin{aligned} \Omega_k &= d_1 w_k + d_2 w_{k-1} + \dots + d_n w_{k-n+1} + v_k \\ \Omega_{k+1} &= d_1 w_{k+1} + d_2 w_k + \dots + d_n w_{k-n+2} + v_{k+1} \\ &\vdots \\ \Omega_{k+n} &= d_1 w_{k+n} + d_2 w_{k-n+1} + \dots + d_n w_{k+1} + v_{k+n} \end{aligned} \quad (3.8)$$

From Equation (3.8) it is apparent that the noise element  $\Omega_k$  is correlated with the elements  $\Omega_{k+1}$ ,  $\Omega_{k+2}$ , ...,  $\Omega_{k+n-1}$ ; however,  $\Omega_k$  and  $\Omega_{k+n}$  are independent with respect to each other. This property of the noise elements is the basis for the identification algorithm. As shown in Appendix A, a set of autocorrelation functions for the noise elements may be defined such that

$$R(0) = E [\Omega_k^2] = \sigma_w^2 (d_1^2 + d_2^2 + \dots + d_n^2) + \sigma_v^2 \quad (3.9)$$

$$R(j) = E [\Omega_k \Omega_{k+1}] = \sigma_w^2 \sum_{i=1}^{n-j} d_i d_{i+j} \quad j = 1, 2, \dots, n-1$$

$$R(j) = 0 \quad j \geq n$$

where  $\sigma_w$  is the standard deviation of the gaussian white sequence of noise inputs  $w_k$  and  $\sigma_v$  is the standard deviation of the measurement noise.

By rearranging Equation (3.5) such that

$$\Omega_k = z_{k+1}^T \underline{\vartheta}_k \quad (3.10)$$

the noise element is now a linear function of the system outputs  $z_{k-n+1}, z_{k-n+2}, \dots, z_{k+1}$  and is therefore known. Hence, the autocorrelation functions of equation (3.9) may be found by taking the statistics  $E[\Omega_k^2], E[\Omega_k \Omega_{k+j}]$ . Equation (3.9) now represents a set of  $n$  simultaneous nonlinear algebraic equations which may be solved for the values of  $d_1, d_2, \dots, d_n$ . The values of  $b_1, b_2, \dots, b_n$  may then be found from equation (3.6).

The general procedure for the identification of the input distribution matrix in a linear system with a random forcing function may be outlined as follows:

1. Measure a scalar linear combination of the states of the system plus additive noise.
2. Identify the canonic form of the state transition matrix of the system.
3. Calculate the values of the noise element  $\Omega_k$  at each sampling time.
4. Calculate the autocorrelation functions  $R(j)$  for  $j = 0, 1, \dots, n-1$ .
5. Solve the simultaneous equations (3.9) for the elements of  $\underline{d}$ .
6. Solve equation (3.6) for the elements of  $\underline{b}$ .

Although the above algorithm does provide a means for identifying the input distribution matrix, two problems arise in the identification process. The first of these involves deciding when to start the identification process. Because the autocorrelation functions are computed by taking the average of the values of  $\Omega_k^2, \Omega_k \Omega_{k+1}, \dots, \Omega_k \Omega_{k+n-1}$  at a number of sampling instants, this process must not begin until enough time has elapsed so that the sample mean and variance of

the noise input and the measurement noise approach 0,  $\sigma_w^2$  and 0,  $\sigma_v^2$  respectively. If the identification process is begun too soon, the computed values of  $\underline{b}$  will be in error from the actual values.

The other problem in the identification process can be best shown by an example. Assuming  $n = 2$ , the autocorrelation functions then become

$$\begin{aligned} R(0) &= \sigma_w^2 (d_1^2 + d_2^2) + \sigma_v^2 \\ R(1) &= \sigma_w^2 d_1 d_2 \end{aligned} \quad (3.11)$$

Solution of Equation (3.11) gives

$$\begin{aligned} d_1 &= \pm \left[ \frac{1}{2\sigma_w^2} \left[ R(0) - \sigma_v^2 \pm \left( [R(0) - \sigma_v^2]^2 - 4[R(1)]^2 \right)^{1/2} \right] \right]^{1/2} \\ d_2 &= \pm \left[ \frac{1}{2\sigma_w^2} \left[ R(0) - \sigma_v^2 \mp \left( [R(0) - \sigma_v^2]^2 - 4[R(1)]^2 \right)^{1/2} \right] \right]^{1/2} \end{aligned} \quad (3.12)$$

and from Equation (3.6)

$$\begin{aligned} b_1 &= d_1 \\ b_2 &= d_2 + \varphi_2 b_1 \end{aligned} \quad (3.13)$$

It is apparent from Equation (3.12) that the identification of the elements  $b_1, b_2$  involves a decision as to which combination of signs to use as the correct one. For higher order systems this decision becomes more complex since the number of possible sign combinations increases.

#### IV. KALMAN FILTER APPROACH

An alternative to the identification algorithm of the preceding section may be derived from the equations of the Kalman filter described below. Consider again a system with the canonic state equations of the form

$$y_{k+1} = \Phi^* y_k + \Gamma^* w_k \quad (4.1)$$

$$z_{k+1} = H^* y_{k+1} + v_{k+1}$$

where  $w_k$  is a scalar random forcing function. The canonic state vector  $y_k$  may be estimated by the technique described by Kalman [3] where, given  $k$  previous measurements, the estimate  $\hat{y}_{k/k}$  at the  $k^{\text{th}}$  sampling instant is given by

$$\hat{y}_{k/k} = \Phi^* \hat{y}_{k-1/k-1} + G_k [z_k - H^* \Phi^* \hat{y}_{k-1/k-1}] \quad (4.2)$$

where  $G_k$  is the filter weighting or gain applied at the  $k^{\text{th}}$  iteration and  $H^*$  is the  $l \times n$  known measurement matrix. The gain is calculated from a set of recursive equations of the form

$$\begin{aligned} G_k &= P_{k/k-1} H^{*T} [H^* P_{k/k-1} H^{*T} + R]^{-1} \\ P_{k/k} &= (I - G_k H^*) P_{k/k-1} \\ P_{k+1/k} &= \Phi^* P_{k/k} \Phi^{*T} + Q \end{aligned} \quad (4.3)$$

where  $P_{k/k}$  and  $P_{k+1/k}$  are the estimation and prediction covariances, respectively, and  $R$  and  $Q$  are given by

$$\begin{aligned} R &\triangleq E [v_k^2] \\ Q &\triangleq \Gamma^* E [w_k^2] \Gamma^{*T} \end{aligned} \quad (4.4)$$

Since all elements necessary to predict the canonic state vector  $\underline{y}_k$  are known except the input distribution matrix  $\Gamma^*$ , the system model based on this estimation scheme will produce suitable estimates for  $\underline{y}_k$  only if the correct  $\Gamma^*$  is used in the filter equations. In order to determine if the correct  $\Gamma^*$  is being used, it is necessary to define a performance index to compare the performance of the model with that of the system. Because the actual system output is available only in the form of the measurement  $z_k$ , the performance index must be defined in terms of  $z_k$  and the predicted measurement  $\hat{z}_{k/k-1}$  as given by

$$\hat{z}_{k/k-1} = H^* \Phi^* \hat{y}_{k-1/k-1}, \quad (4.5)$$

which is obtained from the system model.

Before defining the performance index to be used, it is necessary to define some other terms. The filter is said to have reached the steady-state condition when the prediction covariance matrix does not change from one sampling instant to the next. Under the steady-state condition the actual prediction measurement variance is given by

$$EZ(M) = \frac{1}{M} \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \quad (4.6)$$

where  $K$  is the value of  $k$  at the sampling instant when the filter reaches the steady-state condition. The integer  $M$  is chosen such that the relation

$$|EZ(M+1) - EZ(M)| \leq 0.05 |EZ(M)| \quad (4.7)$$

is valid. The predicted measurement variance is defined as

$$PZ \triangleq E[(z_k - \hat{z}_{k/k-1})^2] \quad (4.8)$$

which, as shown in Appendix B, may be written as

$$PZ = H^* P_K H^{*T} + R \quad (4.9)$$

where  $P_K$  is the prediction covariance under the steady-state conditions of the filter.

From the above definitions the performance index  $J$  is defined as the difference between the actual and the predicted measurement variances and may be written as

$$J = EZ(M) - PZ$$

$$J = \left[ \frac{1}{M} \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \left[ H^* P_K H^{*T} + R \right] \quad (4.10)$$

Since it is desired that the output of the model  $\hat{z}_{k/k-1}$  and the output of the system  $z_k$  be equal for perfect operation of the model,  $J$  is to be minimized. Because both  $EZ(M)$  and  $PZ$  are functions of the input distribution matrix  $\Gamma^*$ , the minimum for  $J$  will result when the correct  $\Gamma^*$  is used in the model. The problem may now be stated for this approach as identifying  $\Gamma^*$  so as to minimize the performance index  $J$ .

Because it would waste time and effort, a trial-and-error search for the correct  $\Gamma^*$  is undesirable. Therefore, it is necessary to derive some method of changing the elements of  $\Gamma^*$  used in the model after each calculation of  $J$ , based on the value of  $J$ . The method investigated is based on a gradient search where the elements of  $\Gamma^*$  are changed by some amount according to the values of a sensitivity function and  $J$ . The sensitivity function must reflect the change in  $J$  caused by changing  $\Gamma^*$ . The method of changing  $\Gamma^*$  may be written as

$$\Gamma_{\text{new}}^* = \Gamma_{\text{old}}^* + \underline{f}(J, \Gamma^*) J \quad (4.11)$$

where the subscripts "old" and "new" represent the value of  $\Gamma^*$  just used in the model and the value to be tried next, respectively. The sensitivity function is denoted by  $\underline{f}(J, \Gamma^*)$  where  $\underline{f}$  is a vector function.

The problem is now to find a suitable sensitivity function to use. Two functions and their failures during simulation are presented in the next section. Because the performance index is a function of the measurements  $z_K, z_{K+1}, \dots, z_{K+M}$  (which are functions of all past measurements) and the predicted measurements during the interval  $K, K+M$ , (which are also functions of all past measurements), the problem of finding a suitable sensitivity function is a complicated one. At present no suitable function has been found.



## V. COMPUTER SIMULATIONS

In order to determine the accuracy of the proposed identification algorithm, a second-order system was simulated on the IBM 360/67 computer. For the system chosen the canonic state transition matrix, the input distribution matrix, and the measurement matrix were

$$\begin{aligned}\Phi^* &= \begin{bmatrix} 0 & 1 \\ -1/16 & 1/2 \end{bmatrix} \\ \Gamma^* &= \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \\ H^* &= \begin{bmatrix} 1 & 0 \end{bmatrix} .\end{aligned}\tag{5.1}$$

In the simulation, the actual and the canonic-state transition matrices were assumed equal so that no error in the identification process would result from a difference in the two, and also for simplicity. The random forcing function and the measurement noise were obtained from a random number generator which gives a gaussian white sequence of any desired mean and standard deviation. The means and standard deviations for the two sequences were

$$\begin{aligned}E [w_k] &= 0 \\ E [w_k^2] &= 2 \\ E [v_k] &= 0 \\ E [v_k^2] &= 1 .\end{aligned}\tag{5.2}$$

The system dynamics were simulated and the measurements  $z_k$  were used to obtain the autocorrelation functions  $R(0)$  and  $R(1)$ . These values



were then used to compute the values of  $b_1$  and  $b_2$  at each sampling instant from the equations

$$\begin{aligned} b_1(k) &= \left[ 0.125 \left[ R(0) - 1 + [(R(0) - 1)^2 - 4R(1)^2]^{1/2} \right] \right]^{1/2} \\ b_2(k) &= \left[ 0.125 \left[ R(0) - 1 - [(R(0) - 1)^2 - 4R(1)^2]^{1/2} \right] \right]^{1/2} \end{aligned} \quad (5.3)$$

The system and the identification process were simulated for 1000 sampling intervals of period one second. An ensemble of 200 simulations was run and an ensemble average for  $R(0)$ ,  $R(1)$ ,  $b_1(k)$ , and  $b_2(k)$  was taken. The results at various sampling intervals are shown in Table I. The ensemble averages of the differences between the actual values of  $\Gamma^*$  and the computed values are also shown.

As can be seen from Table I, there is a bias in the autocorrelation function  $R(1)$  which produces a bias in the estimates for  $b_1$  and  $b_2$ . No explanation for the bias can be given. Simulation of another, similar second-order system also produced a bias, equal to -1.0. For the algorithm to produce the desired results, it is necessary to eliminate the bias.

A second-order system was also simulated for the Kalman-filter approach to the identification problem. The values used for the simulation were

$$\begin{aligned} \Phi^* &= \begin{bmatrix} 0 & 1 \\ 1/8 & 1/4 \end{bmatrix} \\ \Gamma^* &= \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \\ H^* &= \begin{bmatrix} 1 & 0 \end{bmatrix} . \end{aligned} \quad (5.4)$$

The random forcing function and the measurement noise were generated as described above with

$$\begin{aligned}
 E [w_k] &= 0 \\
 E [w_k^2] &= 3 \\
 E [v_k] &= 0 \\
 E [v_k^2] &= R = 4 \quad .
 \end{aligned}
 \tag{5.5}$$

The performance index  $J$  was calculated for various values of  $\Gamma^*$  with the results given in Table II. It can be seen that  $J$  is a minimum when the correct  $\Gamma^*$  is used.

Two sensitivity functions were also calculated during the simulation. The first as derived in Appendix C is given by

$$\underline{f}_1 (J, \Gamma^*) = \frac{\partial PZ}{\partial \Gamma^*} \quad . \tag{5.6}$$

The second, also derived in Appendix C, is given by

$$\underline{f}_2 (J, \Gamma^*) = H^* \frac{\partial G}{\partial \Gamma^*} \left[ \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \frac{\partial PZ}{\partial \Gamma^*} \quad . \tag{5.7}$$

As shown in Table II, neither of these sensitivity functions is suitable since neither produces a consistent pattern by which to change  $\Gamma^*$ . The failure of these functions appears to lie in the fact that neither measures the sensitivity of the measurements to the values of  $\Gamma^*$ .

<u>k</u>	<u>b<sub>1</sub>(k)</u>	<u>1-b<sub>1</sub>(k)</u>	<u>b<sub>2</sub>(k)</u>	<u>1/2-b<sub>2</sub>(k)</u>	<u>E[Ω<sub>k</sub><sup>2</sup>]<sub>k</sub></u>	<u>E[Ω<sub>k</sub>Ω<sub>k+1</sub>]<sub>k</sub></u>
2	0.0	0.0	0.0	0.0	5.11	0.0
10	0.84	0.16	0.65	-0.15	5.25	-0.31
100	1.003	-0.003	0.645	-0.145	5.17	-0.49
200	1.009	-0.009	0.635	0.135	5.18	-0.50
300	1.010	-0.01	0.633	-0.133	5.17	-0.50
400	1.012	-0.013	0.633	-0.133	5.18	-0.50
500	1.016	-0.016	0.631	-0.131	5.20	-0.50
600	1.017	-0.017	0.634	-0.134	5.21	-0.51
700	1.016	-0.016	0.633	-0.133	5.20	-0.51
800	1.018	-0.018	0.634	-0.135	5.22	-0.51
900	1.019	-0.019	0.636	-0.136	5.23	-0.51
999	1.019	-0.019	0.635	-0.135	5.23	-0.51

$$R(0) = \sigma_w^2 (d_1^2 + d_2^2) + \sigma_v^2 = 5.0 \text{ (should match } E[\Omega_k^2])$$

$$R(1) = \sigma_w^2 d_1 d_2 = 0.0 \text{ (should match } E[\Omega_k \Omega_{k+1}])$$

TABLE I. Results of Simulation  
of Identification Algorithm

<u><math>b_1, b_2</math></u>	<u>EZ (M)</u>	<u>PZ</u>	<u>J</u>	<u><math>f_1 (J, \Gamma^*)</math></u>	<u><math>f_2 (J, \Gamma^*)</math></u>
1/2, 1/4	14.7	6.7	8.7	-8.4, -2.3	-2.4, -0.6
1, 1/4	14.6	13.4	1.2	-17.7, -0.3	12.1, 0.2
3/2, 1/4	15.0	24.8	-9.8	-26.6, 0.4	24.1, -0.3
1/2, 1/2	14.1	7.4	6.7	-6.3, -4.2	0.1, 0.04
*1, 1/2	13.7	13.3	0.4	-18.0, 1.1	12.5, -0.8
3/2, 1/2	14.3	24.5	-11.2	-27.3, 2.1	24.7, -1.9
1/2, 3/4	15.7	8.5	7.2	-0.3, -7.6	0.1, 1.3
1, 3/4	19.9	11.0	8.9	-60.5, 60.6	23.0, -23.0
3/2, 3/4	15.0	23.0	-8.0	-31.6, 8.9	28.1, -7.9

\*Correct  $\Gamma^*$

TABLE II. Results of Simulation of  
Kalman Filter Approach

## VI. CONCLUSIONS

The identification of the input distribution matrix of a linear system operating in a stochastic environment is a complicated process, but it can be done to some degree by the proposed algorithm. Further investigation is needed to determine the reason for the bias in the autocorrelation function  $R(1)$ . A general procedure for choosing the proper signs to use in the solution of the nonlinear algebraic equations is also desirable.

The Kalman-filter approach would be an excellent method to use in the identification process if a suitable sensitivity function can be found. This function must reflect the sensitivity of the performance index to the changes in the input distribution matrix. A possible first step in finding the function is to examine the sensitivity of the measurements to the elements of the matrix.

# APPENDIX A

## DERIVATION OF THE AUTOCORRELATION FUNCTIONS FOR THE NOISE ELEMENT

$$R(0) = E[\Omega_k^2]$$

$$\Omega_k \triangleq d_1 w_k + d_2 w_{k-1} + \dots + d_n w_{k-n+1} + v_k$$

$$R(0) = E[(d_1 w_k + d_2 w_{k-1} + \dots + d_n w_{k-n+1})^2]$$

$$\begin{aligned} R(0) = E[ & d_1^2 w_k^2 + d_2^2 w_{k-1}^2 + \dots + d_n^2 w_{k-n+1}^2 + v_k^2 \\ & + 2d_1 d_2 w_k w_{k-1} + 2d_1 d_3 w_k w_{k-2} + \dots + 2d_1 d_n w_k w_{k-n+1} \\ & + 2d_2 d_3 w_{k-1} w_{k-2} + \dots + 2d_2 d_n w_{k-1} w_{k-n+1} \\ & + \dots + 2d_1 w_k v_k + 2d_2 w_{k-1} v_k + \dots + 2d_n w_{k-n+1} v_k] \end{aligned}$$

$$E[w_{k-i}^2] \triangleq \sigma_w^2 \quad i = 0, 1, \dots$$

$$E[v_k^2] \triangleq \sigma_v^2$$

$$E[w_{k-i} w_{k-j}] \triangleq 0 \quad i \neq j$$

$$E[w_{k-i} v_k] \triangleq 0 \quad i = 0, 1, \dots$$

$$R(0) = E[d_1^2 w_k^2] + E[d_2^2 w_{k-1}^2] + \dots + E[d_n^2 w_{k-n+1}^2] + E[v_k^2]$$

$$R(0) = d_1^2 E[w_k^2] + d_2^2 E[w_{k-1}^2] + \dots + E[d_n^2 w_{k-n+1}^2] + \sigma_v^2$$

$$R(0) = \sigma_w^2 (d_1^2 + d_2^2 + \dots + d_n^2) + \sigma_v^2$$

$$R(j) = E[\Omega_k \Omega_{k+j}]$$

$$R(j) = E[(d_1 w_k + d_2 w_{k-1} + \dots + d_n w_{k-n+1} + v_k) \cdot$$

$$(d_1 w_{k+j} + d_2 w_{k+j-1} + \dots + d_n w_{k-n+j+1} + v_{k+j})]$$

$$\begin{aligned}
R(j) = & E[d_1^2 w_k w_{k+j} + d_2^2 w_{k-1} w_{k+j-1} + \dots + d_n^2 w_{k-n+1} w_{k-n+j+1} \\
& + d_1 d_2 w_k w_{k+j-1} + d_1 d_3 w_k w_{k+j-2} + \dots + d_1 d_n w_k w_{k-n+j+1} \\
& + d_1 d_2 w_{k-1} w_{k+j} + d_2 d_3 w_{k-1} w_{k+j-2} + \dots + d_2 d_n w_{k-1} w_{k-n+j+1} \\
& + \dots + d_1 d_n w_{k+j} w_{k-n+1} + d_2 d_n w_{k+j-1} w_{k-n+1} + \dots \\
& + d_{n-1} d_n w_{k-n+1} w_{k-n+j} + v_{k+j} (d_1 w_k + d_2 w_{k-1} + \dots \\
& + d_n w_{k-n+1}) + v_k (d_1 w_{k+j} + d_2 w_{k+j-1} + \dots + d_n w_{k-n+j+1}) \\
& + v_k v_{k+j}]
\end{aligned}$$

$$E[w_{k-i} w_{k+j-l}] \stackrel{\Delta}{=} \sigma_w^2 \quad i = l-j \quad \text{and} \quad j = 1, 2, \dots, n-1$$

$$E[w_{k-i} w_{k+j-l}] \stackrel{\Delta}{=} 0 \quad i \neq l-j \quad \text{or} \quad j \geq n$$

$$E[v_k v_{k+j}] \stackrel{\Delta}{=} 0$$

$$E[v_k w_{k+j}] \stackrel{\Delta}{=} 0 \quad j = 0, 1, \dots$$

$$R(1) = E[d_1 d_2 w_k^2 + d_2 d_3 w_{k-1}^2 + \dots + d_{n-1} d_n w_{k-n+1}^2]$$

$$R(1) = \sigma_w^2 (d_1 d_2 + d_2 d_3 + \dots + d_{n-1} d_n)$$

$$R(2) = E[d_1 d_3 w_k^2 + d_2 d_4 w_{k-1}^2 + \dots + d_{n-2} d_n w_{k-n+1}^2]$$

$$R(2) = \sigma_w^2 (d_1 d_3 + d_2 d_4 + \dots + d_{n-2} d_n)$$

⋮

$$R(n-1) = E[d_1 d_2 w_k^2]$$

$$R(n-1) = \sigma_w^2 d_1 d_n$$

$$R(j) = 0 \quad j \geq n$$

$$R(j) = \sigma_w^2 \sum_{i=1}^{n-j} d_j d_{j+i} \quad j = 1, 2, \dots, n-1$$

$$R(j) = 0 \quad j \geq n$$

## APPENDIX B

### DERIVATION OF THE PREDICTED MEASUREMENT VARIANCE

$$z_k = H^* y_k + v_k$$

$$\hat{z}_{k/k-1} = H^* \hat{\Phi}^* \hat{y}_{k-1/k-1} = H^* \hat{y}_{k/k-1}$$

$$(z_k - \hat{z}_{k/k-1})^2 = [H^* y_k + v_k - H^* \hat{y}_{k/k-1}]^2$$

$$PZ \stackrel{\Delta}{=} E[(z_k - \hat{z}_{k/k-1})^2]$$

$$PZ = E[[H^*(y_k - \hat{y}_{k/k-1}) + v_k]^2]$$

$$PZ = E[[H^*(y_k - \hat{y}_{k/k-1}) + v_k] [(y_k - \hat{y}_{k/k-1})^T H^{*T} + v_k]]$$

$$PZ = E[H^* (y_k - \hat{y}_{k/k-1}) (y_k - \hat{y}_{k/k-1})^T H^{*T} + v_k^2 \\ + H^* (y_k - \hat{y}_{k/k-1}) v_k + v_k (y_k - \hat{y}_{k/k-1})^T H^{*T}]$$

$$E[v_k^2] \stackrel{\Delta}{=} R$$

$$E[v_k] \stackrel{\Delta}{=} 0$$

$$E[(y_k - \hat{y}_{k/k-1}) (y_k - \hat{y}_{k/k-1})^T] \stackrel{\Delta}{=} P_K \text{ (in steady-state)}$$

$$PZ = H^* P_K H^{*T} + R$$



## APPENDIX C

### DERIVATIONS OF THE SENSITIVITY FUNCTIONS

The first sensitivity function to be derived is  $\frac{\partial PZ}{\partial \Gamma^*}$  during steady-state operation of the filter. This is done by finding an expression for the prediction covariance during steady-state operation, equating elements on both sides of the equation, and then taking the desired partial derivatives of the implicit functions. The derivation follows.

$$G_k = P_{k/k-1} H^{*T} [H^* P_{k/k-1} H^{*T} + R]^{-1}$$

$$P_{k/k} = (I - G_k H^*) P_{k/k-1}$$

$$P_{k/k} = (I - P_{k/k-1} H^{*T} [H^* P_{k/k-1} H^{*T} + R]^{-1} H^*) P_{k/k-1}$$

$$P_{k+1/k} = \Phi^* P_{k/k} \Phi^{*T} + Q$$

$$P_{k+1/k} = \Phi^* \left[ (I - P_{k/k-1} H^{*T} [H^* P_{k/k-1} H^{*T} + R]^{-1} H^*) P_{k/k-1} \right] \Phi^{*T} + Q$$

DURING STEADY-STATE OPERATION

$$P_{k+1/k} = P_{k/k-1} = P_K$$

$$P_K = \Phi^* \left[ (I - P_K H^{*T} [H^* P_K H^{*T} + R]^{-1} H^*) P_K \right] \Phi^{*T} + Q$$

FOR A SECOND-ORDER SYSTEM

$$P_K = \begin{bmatrix} 0 & 1 \\ \varphi_1 & \varphi_2 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right. \right. \\ \left. \left. + R \right\}^{-1} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \right\} \begin{bmatrix} 0 & \varphi_1 \\ 1 & \varphi_2 \end{bmatrix} \\ + \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

$$P_K = \left[ \begin{array}{c|c} \frac{-P_{21}^2}{P_{11} + R} + P_{22} + Q_{11} & \frac{-P_{21}}{P_{11} + R} (\varphi_1 P_{11} + \varphi_2 P_{12}) + (\varphi_1 P_{21} + \varphi_2 P_{22}) + Q_{12} \\ \hline \frac{-P_{21}}{P_{11} + R} (\varphi_1 P_{11} + \varphi_2 P_{21}) & (\varphi_1 P_{11} + \varphi_2 P_{12}) \left( \varphi_1 - \frac{\varphi_1 P_{11}}{P_{11} + R} - \frac{\varphi_2 P_{21}}{P_{11} + R} \right) \\ \hline + (\varphi_2 P_{21} + \varphi_2 P_{22}) + Q_{21} & + \varphi_2 (\varphi_1 P_{21} + \varphi_2 P_{22}) + Q_{22} \end{array} \right]$$

$$P_{21} = P_{12}$$

$$Q_{21} = P_{12}$$

EQUATING ELEMENTS

$$P_{11} = \frac{-P_{21}^2}{P_{11} + R} + P_{22} + Q_{11}$$

$$P_{21} = P_{12} = \frac{-P_{21}}{P_{11} + R} (\varphi_1 P_{11} + \varphi_2 P_{21}) + (\varphi_2 P_{21} + \varphi_2 P_{22}) + Q_{12}$$

$$P_{22} = (\varphi_1 P_{11} + \varphi_2 P_{21}) \left( \varphi_1 - \frac{\varphi_1 P_{11}}{P_{11} + R} - \frac{\varphi_2 P_{21}}{P_{11} + R} \right) + \varphi_2 (\varphi_1 P_{21} + \varphi_2 P_{22}) + Q_{22}$$

REARRANGING

$$(F.) \quad P_{11}^2 + P_{11} (R - P_{22} - N\gamma_1^2) + (P_{21}^2 - P_{22}R - NR\gamma_1^2) = 0$$

$$(G.) \quad \varphi_2^2 P_{21}^2 + P_{21} (P_{11} + R - \varphi_1 R) - (\varphi_2 P_{22} P_{11} + \varphi_2 R P_{22} + P_{11} N\gamma_1\gamma_2 + NR\gamma_1\gamma_2) = 0$$

$$(H.) \quad P_{22} (P_{11} + R - \varphi_2^2 P_{11} - \varphi_2^2 R) - (\varphi_1^2 P_{11}R - \varphi_2^2 P_{21}^2 + 2\varphi_1 \varphi_2 P_{21}R + P_{11}N\gamma_2^2 + NR\gamma_2^2) = 0$$

$$N = E [w_k^2]$$

$$Q = N \begin{bmatrix} \gamma_1^2 & \gamma_1 \gamma_2 \\ \gamma_1 \gamma_2 & \gamma_2^2 \end{bmatrix}$$

$$\Gamma^* = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$

$$PZ = H^* P_K H^{*T} + R$$

$$PZ = [1 \ 0] \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + R$$

$$PZ = P_{11} + R$$

$$(F.) \quad PZ^2 - PZ (R + P_{22} + N\gamma_1^2) + P_{21}^2 = 0$$

$$(G.) \quad \varphi_2^2 P_{21}^2 + P_{21} (PZ - \varphi_1 R) - (\varphi_2 P_{22} PZ + PZ N\gamma_1 \gamma_2) = 0$$

$$(H.) \quad P_{22} PZ (1 - \varphi_2^2) - \varphi_1^2 PZ R - 2 \varphi_1 \varphi_2 P_{21} R + \varphi_2^2 P_{21}^2 \\ + \varphi_1^2 R^2 - PZ N\gamma_2^2 = 0$$

$$\frac{\partial PZ}{\partial \gamma_1} = \frac{\begin{vmatrix} \frac{\partial F}{\partial \gamma_1} & \frac{\partial F}{\partial P_{21}} & \frac{\partial F}{\partial P_{22}} \\ \frac{\partial G}{\partial \gamma_1} & \frac{\partial G}{\partial P_{21}} & \frac{\partial G}{\partial P_{22}} \\ \frac{\partial H}{\partial \gamma_1} & \frac{\partial H}{\partial P_{21}} & \frac{\partial H}{\partial P_{22}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F}{\partial PZ} & \frac{\partial F}{\partial P_{21}} & \frac{\partial F}{\partial P_{22}} \\ \frac{\partial G}{\partial PZ} & \frac{\partial G}{\partial P_{21}} & \frac{\partial G}{\partial P_{22}} \\ \frac{\partial H}{\partial PZ} & \frac{\partial H}{\partial P_{21}} & \frac{\partial H}{\partial P_{22}} \end{vmatrix}}$$

D

$$\frac{\partial PZ}{\partial \gamma_1} = \frac{\begin{vmatrix} -2N\gamma_1 PZ & 2P_{21} & -PZ \\ -N\gamma_2 PZ & 2\omega_2^2 P_{21} + PZ - \varphi_1 R & -\omega_2 PZ \\ 0 & -2\omega_1 \omega_2 R + 2\omega_2^2 P_{21} & PZ - \omega_2^2 PZ \end{vmatrix}}{\begin{vmatrix} 2PZ - R - P_{22} - N\gamma_1^2 & 2P_{21} & -PZ \\ P_{21} - \omega_2 P_{22} - N\gamma_1 \gamma_2 & 2\omega_2^2 P_{21} + PZ - \varphi_1 R & -\omega_2 PZ \\ P_{22} - \omega_2^2 P_{22} - \varphi_1^2 R^2 - N\gamma_2^2 & -2\omega_1 \omega_2 R + 2\omega_2^2 P_{21} & PZ - \omega_2^2 PZ \end{vmatrix}}$$

$$\frac{\partial PZ}{\partial \gamma_2} = \frac{\begin{vmatrix} \frac{\partial F}{\partial \gamma_2} & \frac{\partial F}{\partial P_{21}} & \frac{\partial F}{\partial P_{22}} \\ \frac{\partial G}{\partial \gamma_2} & \frac{\partial G}{\partial P_{21}} & \frac{\partial G}{\partial P_{22}} \\ \frac{\partial H}{\partial \gamma_2} & \frac{\partial H}{\partial P_{21}} & \frac{\partial H}{\partial P_{22}} \end{vmatrix}}{|D|}$$

$$\frac{\partial PZ}{\partial \gamma_2} = \frac{\begin{vmatrix} 0 & 2P_{21} & -PZ \\ -N\gamma_1 PZ & 2\omega_2^2 P_{21} + PZ - \varphi_1 R & -\omega_2 PZ \\ -2N\gamma_2 PZ & -2\omega_1 \omega_2 R + 2\omega_2^2 P_{21} & PZ - \omega_2^2 PZ \end{vmatrix}}{|D|}$$

$$\frac{\partial PZ}{\partial \Gamma^*} = \begin{bmatrix} \frac{\partial PZ}{\partial \gamma_1} \\ \frac{\partial PZ}{\partial \gamma_2} \end{bmatrix}$$

Another sensitivity function of the form

$$\underline{f}(J, \Gamma^*) = H^* \frac{\partial G_K}{\partial \Gamma^*} \left[ \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \frac{\partial PZ}{\partial \Gamma^*}$$

is to be considered now. It may be found by expressing the filter gain in steady state as a function of the steady-state prediction covariance. The required partial derivatives may then be found as above. The following derivation is done assuming steady-state operation of the filter.

$$G_K = P_K H^{*T} [H^* P_K H^{*T} + R]^{-1}$$

FOR A SECOND-ORDER SYSTEM

$$G_K = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + R \right\}^{-1}$$

$$G_K = \frac{1}{P_{11} + R} \begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix}$$

$$G_K = \frac{1}{PZ} \begin{bmatrix} PZ - R \\ P_{21} \end{bmatrix}$$

$$\underline{f}(J, \Gamma^*) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial (\frac{PZ-R}{PZ})}{\partial \Gamma^*} \\ \frac{\partial (P_{21}/PZ)}{\partial \Gamma^*} \end{bmatrix} \left[ \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \frac{\partial PZ}{\partial \Gamma^*}$$

$$\underline{f}(J, \Gamma^*) = \frac{\partial (\frac{PZ-R}{PZ})}{\partial \Gamma^*} \left[ \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \frac{\partial PZ}{\partial \Gamma^*}$$

$$\underline{f}(J, \Gamma^*) = \left( 1 + \frac{R}{PZ} \frac{\partial PZ}{\partial \Gamma^*} \right) \left[ \sum_{i=K}^{K+M} (z_i - \hat{z}_{i/i-1})^2 \right] - \frac{\partial PZ}{\partial \Gamma^*}$$

where  $\frac{\partial PZ}{\partial \Gamma^*}$  is as given above.

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An algorithm for the identification of the input distribution matrix of a linear, stationary system operating in a stochastic environment is derived. The identification is accomplished by defining a set of auto-correlation functions for a noise element composed of a linear combination of the input distribution matrix elements and the random excitations of the system. Another possible identification method employing a Kalman filter is discussed and the problems associated with its derivation are presented. Results of computer simulations for both methods are included.



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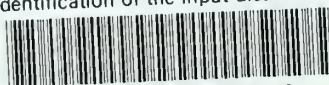






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Identification of the input distribution



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